Applications of Probability Theory

- The subject of probability can be traced back to the 17th century when it arose out of the study of **gambling games**.
- The range of applications extends beyond games into business decisions, insurance, law, **medical tests**, and the social sciences.
- The **stock market**, "the largest casino in the world," cannot do without it.
- The **telephone network**, call centers, and airline companies with their randomly fluctuating loads could not have been economically designed without probability theory.

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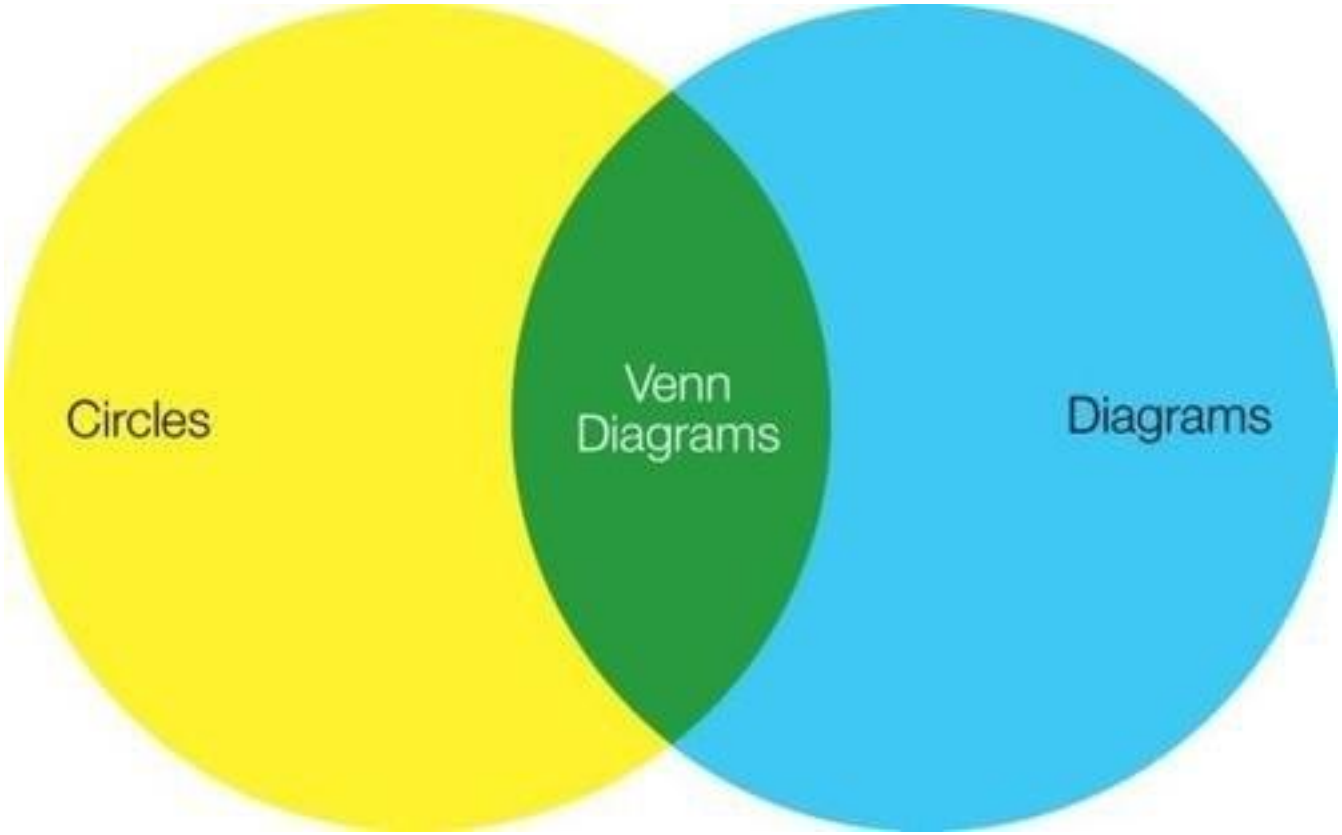
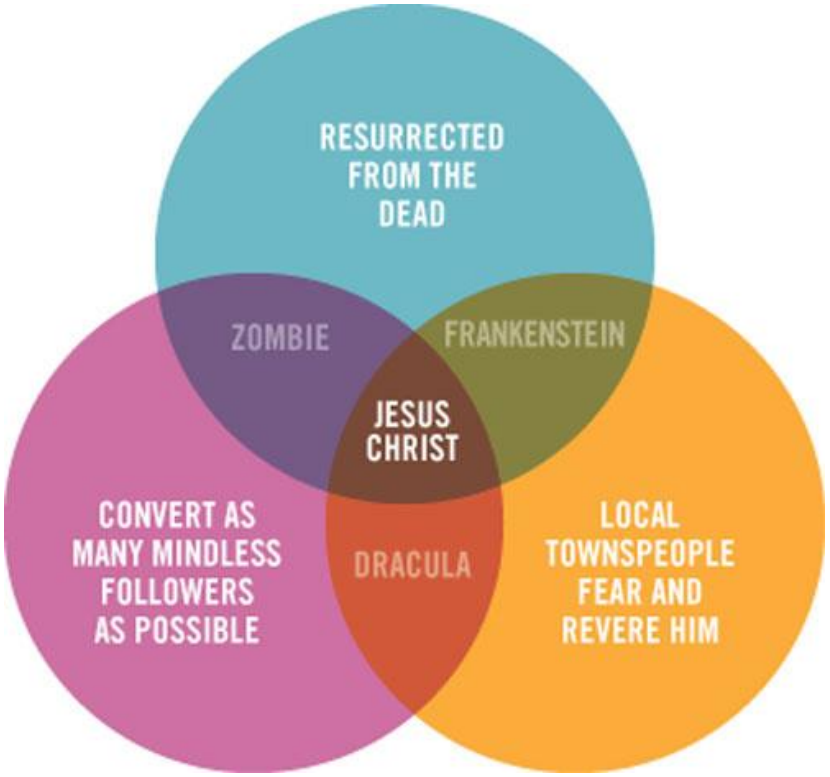
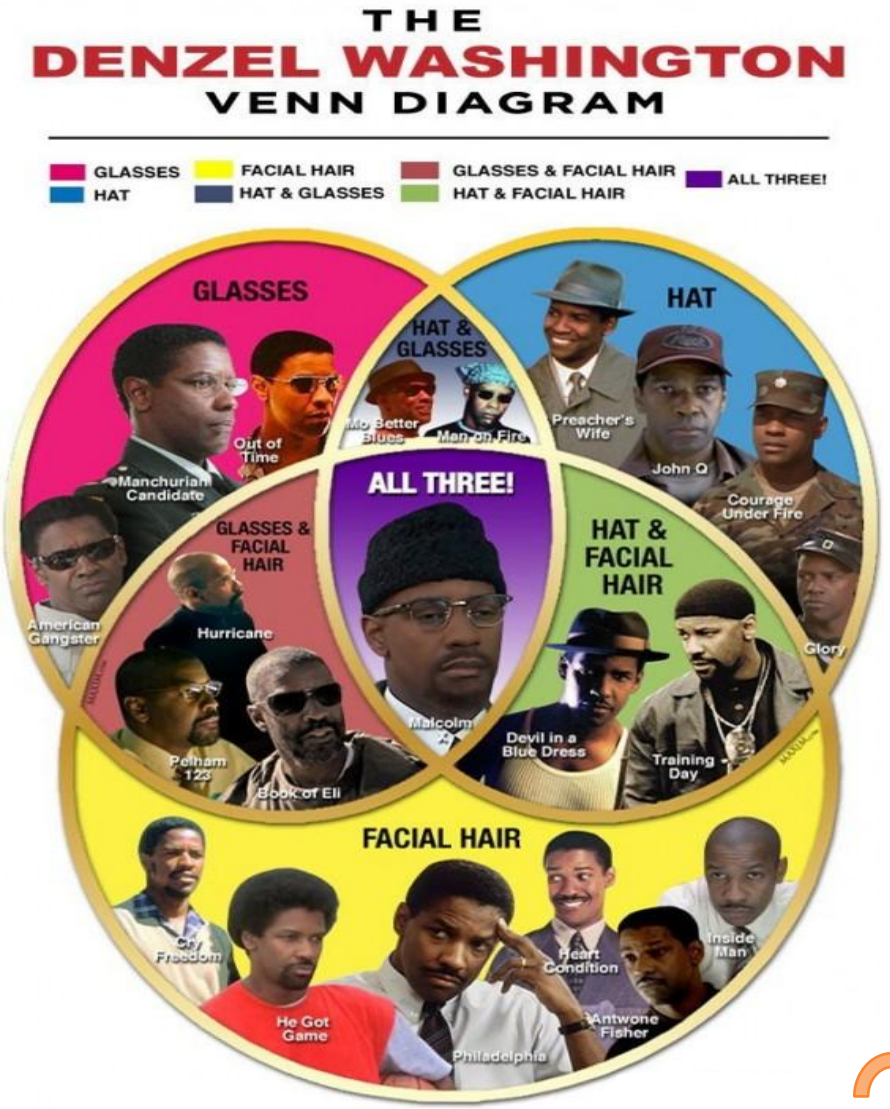
2 Review of Set Theory

Venn diagram



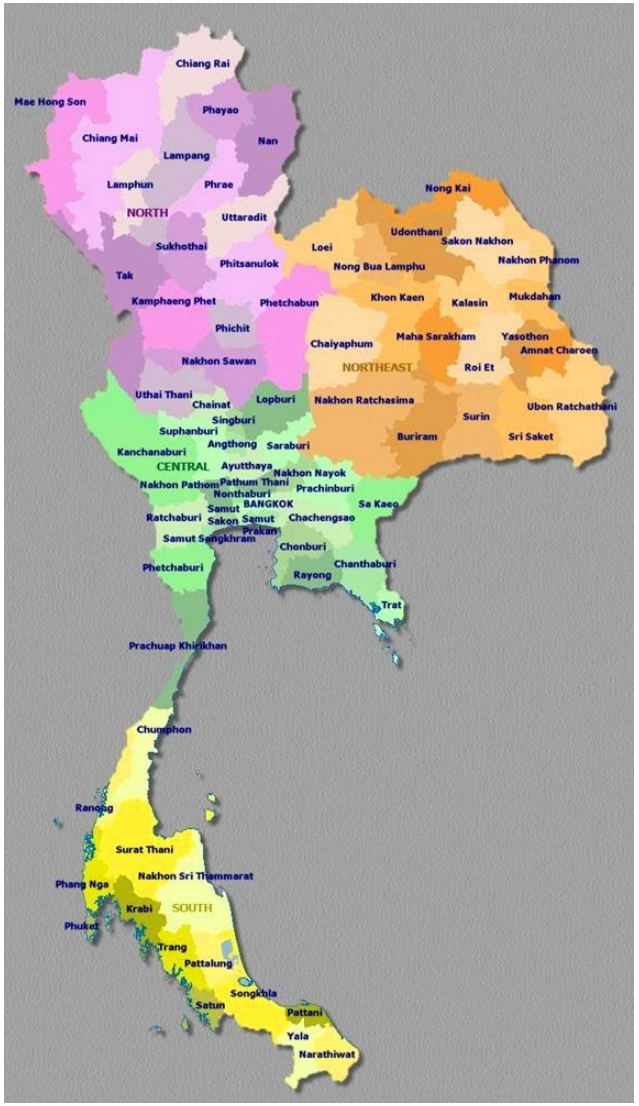
Venn diagram: Examples





Partition


WASHINGTON
OREGON
CALIFORNIA
IDAHO
NEVADA
MONTANA
WYOMING
UTAH
ARIZONA
COLORADO
NEW MEXICO
NORTH DAKOTA
SOUTH DAKOTA
NEBRASKA
KANSAS
OKLAHOMA
TEXAS
MINNESOTA
IOWA
MISSOURI
ARKANSAS
LOUISIANA
WISCONSIN
ILLINOIS
MICHIGAN
INDIANA
OHIO
KENTUCKY
TENNESSEE
MISSISSIPPI
ALABAMA
GEORGIA
FLORIDA
SOUTH CAROLINA
NORTH CAROLINA
VIRGINIA
WEST VIRGINIA
PENNSYLVANIA
NEW YORK
MAINE
VERMONT
NEW HAMPSHIRE
MASSACHUSETTS
RHODE ISLAND
CONNECTICUT
NEW JERSEY
DELAWARE
MARYLAND
Washington D.C.
ALASKA
HAWAII


Chiang Rai
Mae Hong Son
Phayao
Chiang Mai
Nan
Lampang
Lamphun
Phrae
NORTH
Uttaradit
Nong Kai
Udonthani
Sakon Nakhon
Sukhothai
Loei
Nakhon Phanom
Phitsanulok
Nong Bua Lamphu
Tak
Kamphaeng Phet
Phetchabun
Khon Kaen
Kalasin
Mukdahan
Phichit
Chaiyaphum
Maha Sarakham
Yasothon
Amnat Charoen
Nakhon Sawan
NORTHEAST
Roi Et
Uthai Thani
Chainat
Lopburi
Nakhon Ratchasima
Surin
Ubon Ratchathani
Singburi
Suphanburi
Buriram
Sri Saket
Angthong
Saraburi
Kanchanaburi
CENTRAL
Ayutthaya
Nakhon Nayok
Nakhon Pathom
Pathum Thani
Prachinburi
Nonthaburi
BANGKOK
Sa Kaeo
Samut Sakhon
Samut Prakan
Chachengsao
Ratchaburi
Samut Songkhram
Chonburi
Phetchaburi
Chanthaburi
Rayong
Trat
Prachuap Khirikhan
Chumphon
Ranong
Surat Thani
Nakhon Sri Thammarat
Phang Nga
Krabi
SOUTH
Phuket
Trang
Pattalung
Songkhla
Satun
Pattani
Yala
Narathiwat

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3 Classical Probability

Real coins are biased

- From a group of Stanford researchers

DYNAMICAL BIAS IN THE COIN TOSS

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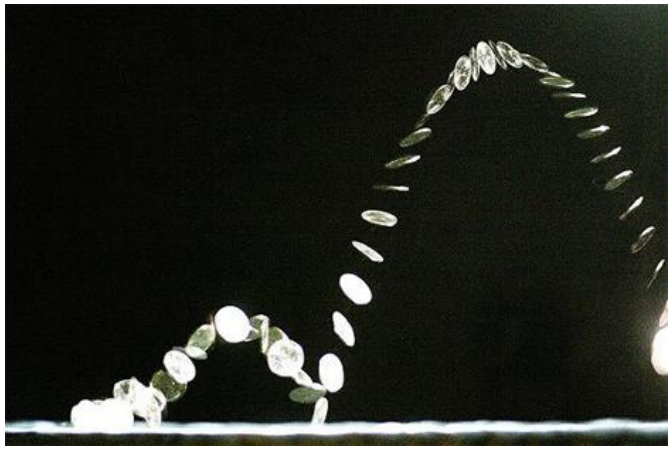
Departments of Mathematics and Statistics
Stanford University

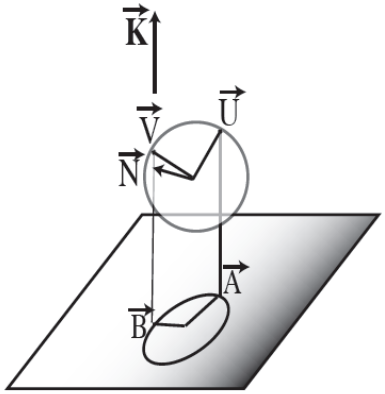
Susan Holmes
Department of Statistics
Sequoia Hall
Stanford University

Richard Montgomery
Department of Mathematics
University of California
Santa Cruz

Abstract

We analyze the natural process of flipping a coin which is caught in the hand. We prove that vigorously-flipped coins are biased to come up the same way they started. The amount of bias depends on a single parameter, the angle between the normal to the coin and the angular momentum vector. Measurements of this parameter based on high-speed photography are reported. For natural flips, the chance of coming up as started is about .51.

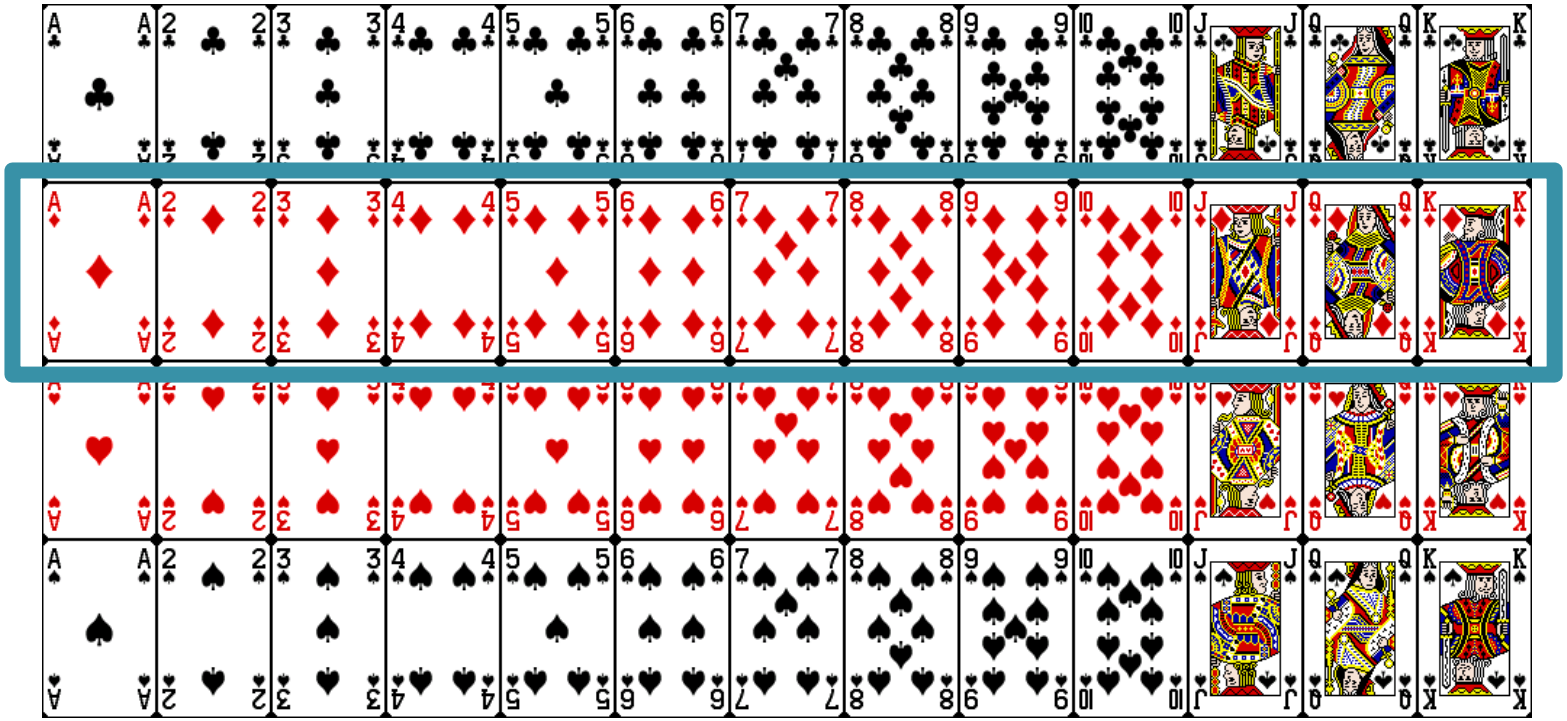




http://gajitz.com/up-in-the-air-coin-tosses-not-as-neutral-as-you-think/
http://www.codingthewheel.com/archives/the-coin-flip-a-fundamentally-unfair-proposition
http://www-stat.stanford.edu/~susan/papers/headswithJ.pdf

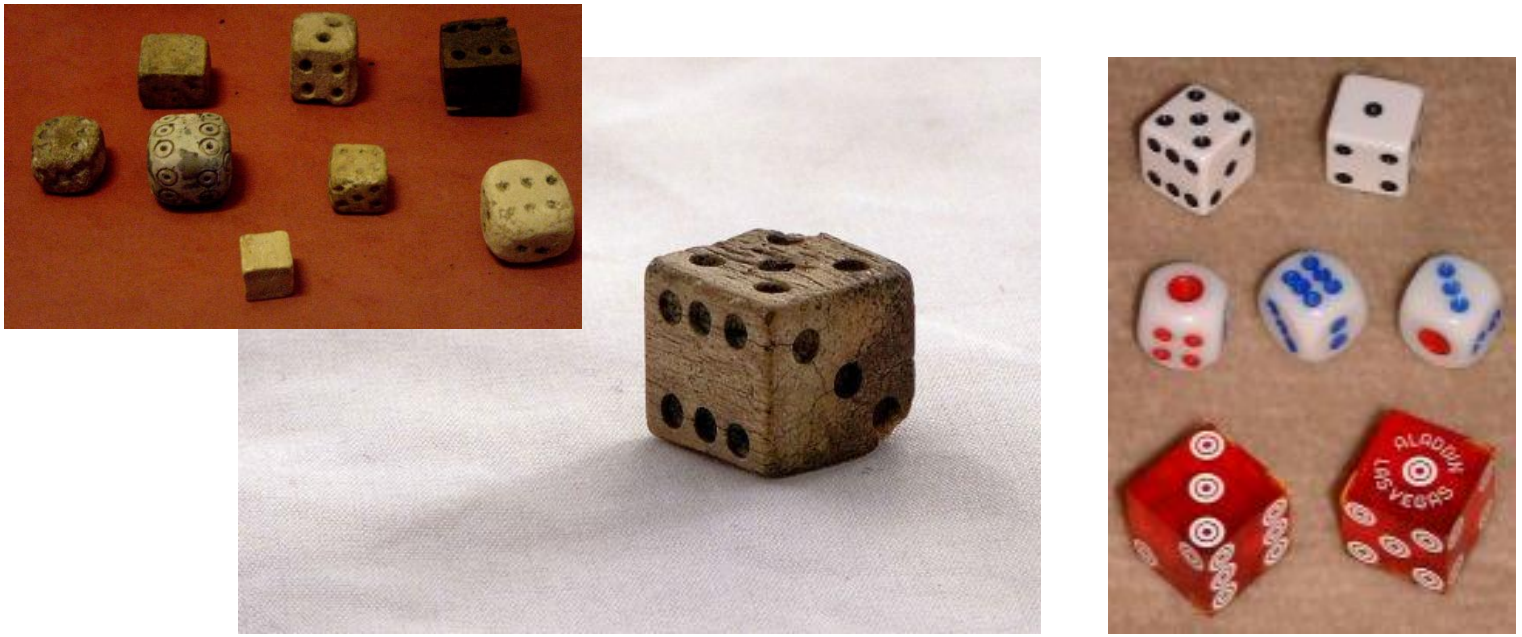
Example

- In drawing a card from a deck, there are 52 equally likely outcomes, 13 of which are **diamonds**. This leads to a probability of 13/52 or 1/4.



The word “dice”

- Historically, **dice** is the plural of **die**.
- In modern standard English, **dice** is used as both the singular and the plural.

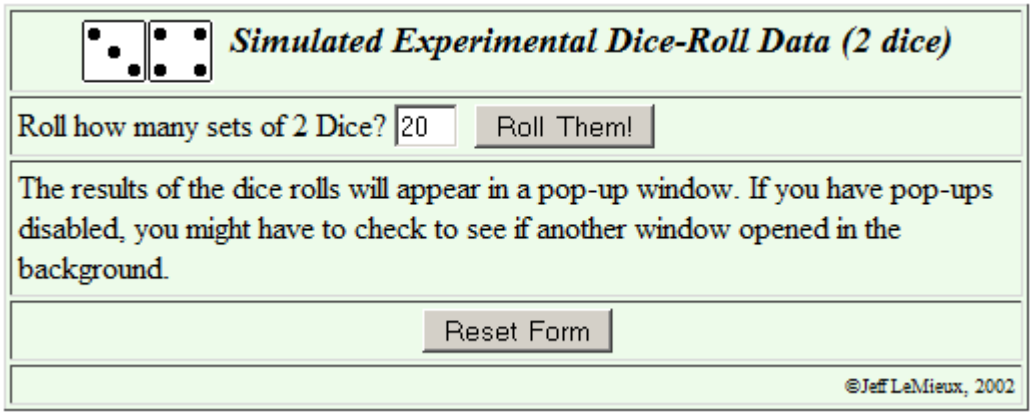


Example of 19th Century bone dice

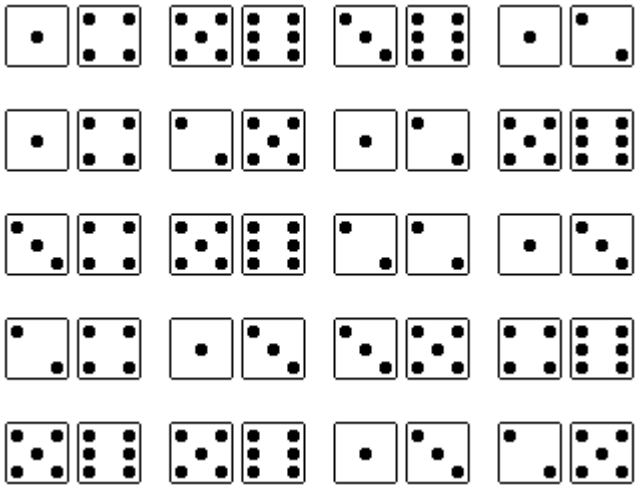
“Advanced” dice

[http://gmdice.com/]

Two dice: Simulation

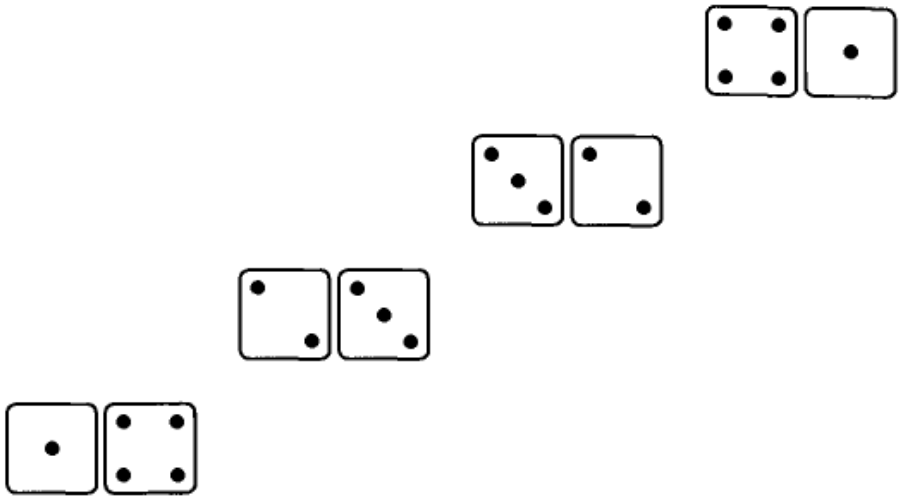


[http://www2.whidbey.net/ohmsmath/webwork/javascript/dice2rol.htm]



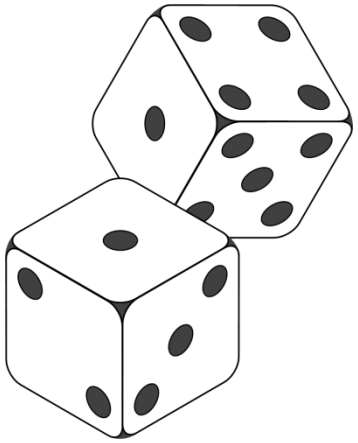
Two dice

- Assume that the two dice are fair and independent.
- P[sum of the two dice = 5] = 4/36



Two dice

- Assume that the two dice are fair and independent.



DICE CHART		
ROLL		PROBABILITY →
2		1/36
3		2/36
4		3/36
5		4/36
6		5/36
7		6/36
8		5/36
9		4/36
10		3/36
11		2/36
12		1/36

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4 Combinatorics

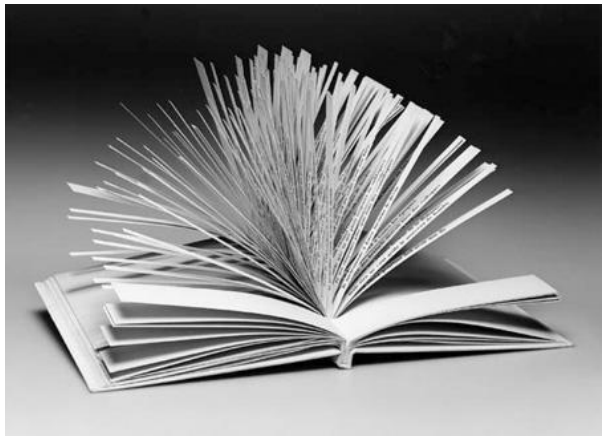
Heads, Bodies and Legs flip-book

Heads, Bodies and Legs flip-book (2)

One Hundred Thousand Billion Poems

- Cent mille milliards de poèmes

One Hundred Thousand Billion Poems (2)



Scandal of Arithmetic

Which is more likely, obtaining at least one six in 4 tosses of a fair die (event A), or obtaining at least one double six in 24 tosses of a pair of dice (event B)?

[http://www.youtube.com/watch?v=MrVD4q1m1Vo]

Scandal of Arithmetic

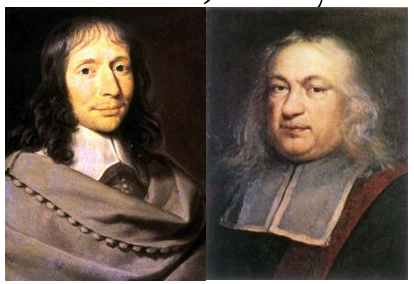
Which is more likely, obtaining at least one six in 4 tosses of a fair die (event A), or obtaining at least one double six in 24 tosses of a pair of dice (event B)?

$$P(A) = \frac{6^4 - 5^4}{6^4} = 1 - \left(\frac{5}{6}\right)^4 \approx .518$$

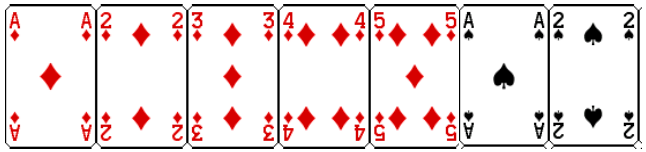
$$P(B) = \frac{36^{24} - 35^{24}}{36^{24}} = 1 - \left(\frac{35}{36}\right)^{24} \approx .491$$

“Origin” of Probability Theory

- Probability theory was originally inspired by **gambling** problems.
- In 1654, Chevalier de Mere invented a gambling system which bet even money on case B on the previous slide.
- When he began losing money, he asked his mathematician friend Blaise **Pascal** to analyze his gambling system.
- Pascal discovered that the Chevalier's system would lose about 51 percent of the time.
- Pascal became so interested in probability and together with another famous mathematician, Pierre de **Fermat**, they laid the foundation of probability theory.



best known for Fermat's Last Theorem



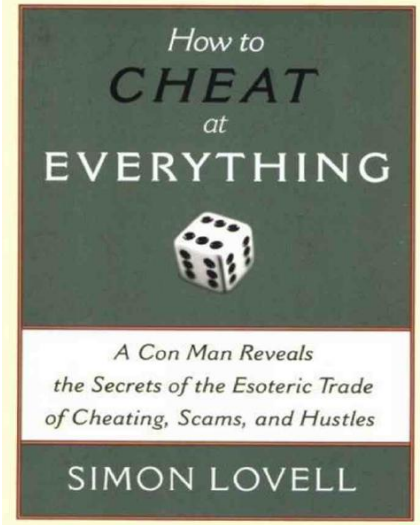
Example: The Seven Card Hustle

- Take five red cards and two black cards from a pack.
- Ask your friend to shuffle them and then, without looking at the faces, lay them out in a row.

- Bet that them can't turn over three red cards.
- The probability that they CAN do it is

$$\frac{\cancel{5}\times 4\times 3}{7\times 6\times \cancel{5}} = \frac{2}{7}$$

$$\frac{\binom{5}{3}}{\binom{7}{3}} = \frac{5!}{\cancel{3}!2!}\times\frac{\cancel{3}!4!}{7!} = 5\times 4\times 3\times\frac{1}{7\times 6\times 5} = \frac{2}{7}$$



[Lovell, 2006]

Finger-Smudge on Touch-Screen Devices

FRUIT NINJA

ANGRY BIRDS

- Fingers' oily smear on the screen
- Different apps gives different finger-smudges.
- Latent smudges may be usable to infer recently and frequently touched areas of the screen--a form of **information leakage**.

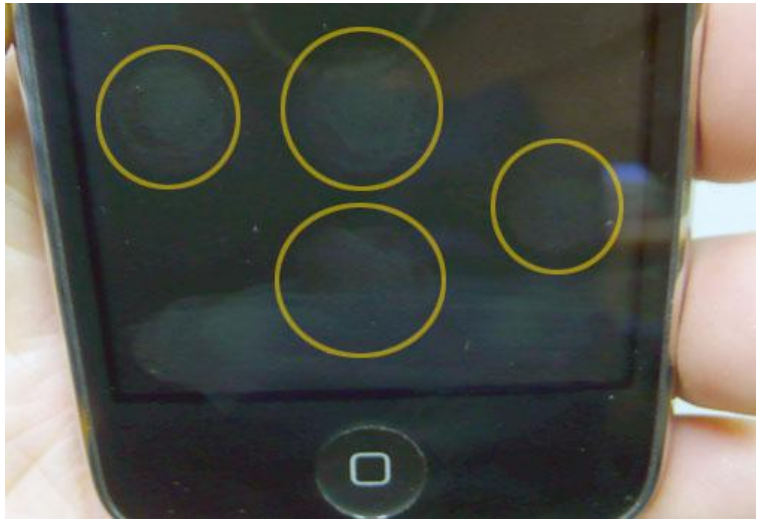
[http://www.ijsmblog.com/2011/02/ipad-finger-smudge-art.html]

Lockscreen PIN / Passcode

[http://lifehacker.com/5813533/why-you-should-repeat-one-digit-in-your-phones-4+digit-lockscreen-pin]

Smudge Attack

- Touchscreen smudge may give away your password/passcode
- Four distinct fingerprints reveals the four numbers used for passcode lock.



[http://www.engadget.com/2010/08/16/shocker-touchscreen-smudge-may-give-away-your-android-password/2]

Suggestion: Repeat One Digit

- Unknown numbers:
  - The number of 4-digit different passcodes $= 10^4$
- Exactly four different numbers:
  - The number of 4-digit different passcodes $= 4! = 24$
- Exactly three different numbers:
  - The number of 4-digit different passcodes $= 3 \times \binom{4}{2} = 36$

Choose the number that will be repeated

Choose the locations of the two non-repeated numbers.